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In an effort to cross-validate a Japanese study, a 16 item test of logical connectives was administered to 223 boys and girls at each grade level (third through ninth grades) and two groups of college sophomore girls. The purpose of the test was to assess their understanding of class inclusion and exclusion, class intersection, and class union. Half of the groups received a test in which set elements were pictures; the other half had words as set elements. Although there were significant differences betwee n grades for all three types of questions, (a) inclusion and exclusion are understood by a majority of even the youngest children, (b) intersect is understood by a majority of all bu t the youngest children, and (c) union is not understood by the majority of subjects except at the college level. Those taking the test, in which set elements were pictures, performed better than those taking the test in which set elements were words. These results, in general, support the findings of the Japanese study, although Japanese children as a group scored higher than American children. The most complete explanation of the present data seems to be an analysis of performance in terms of component operations for processing and storing information. (MH)

Development of the Understanding of Logical Connectives

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Although language is assumed to play a key role in thinking, especially at the more abstract levels, there is amazingly little evidence on development in understanding of such fundamental language elements as quantifiers (all, some) and connectives (or, and) in the context of logical statements. One notable study of logical connectives by Nitta and Nagano (1963) is only partially reported in English (Nitta and Nagano, 1966). They administered ten different versions of a 16 item test to 679 children in grades K, 2, 4, 6, and 8 over several sessions in order to study age changes in the interpretation of class inclusion (A, B), exclusion ($\bar{A}$, $\bar{B}$), intersection (A and B), and union (A or B). They found that even the youngest children correctly answered inclusion and exclusion questions; intersection was difficult for the youngest children but well understood by older children; union, on the other hand, is difficult even for older children.[2] The present study employed an English translation of two of their tests with American children to determine the generality of their findings to a different language group.

A mimeographed test of 16 items, each followed by eight alternatives, was prepared in two forms: in one the alternatives were pictures of black or white birds or flowers, in the second the alternatives were the names of eight common objects. In each case

S was to circle all of the alternatives described by the statement which preceded them. The first four statements dealt with class inclusion and exclusion: A, $\bar{A}$, B, $\bar{B}$; the next four statements involved intersection of classes: A and B, $\bar{A}$ and $\bar{B}$, $\bar{A}$ and B, A and $\bar{B}$; the remaining eight involved class union: A or B, $\bar{A}$ or $\bar{B}$, $\bar{A}$ or B, A or $\bar{B}$. The first four of these were phrased in the form "A or B or both" to clarify the inclusive interpretation of "or", while the last four used an unqualified "A or B". The picture version employed as elements the eight pictures used by Nitta and Nagano in their test 7: intersecting sets in which A=birds, $\bar{A}$=flowers, B=black, and $\bar{B}$=white. The eight elements were two black and two white birds and two black and two white flowers; the two members of each instance differed with respect to the irrelevant dimension of size. The word version was a modification of Nitta and Nagano test 9 which, again, involved intersecting sets: A=flying things and B=living things. The eight elements were Airplane, Bee, Warship, Goldfish, Kite, Sparrow, Tricycle, and Elephant. The tests were group-administered to intact classes; each child did only one form of the test.

Method

Each author tested two classes at each grade level; procedure differed slightly as appropriate with the grade level. The first author tested 6, 7, and 8 grade mathematics classes in the Highland Park Middle School and two introductory psychology discussion sections at Douglass College. The second author tested 3, 4, 5, and 6 graders at Grandview Elementary School and 7, 8, and 9 graders at Conackamack Junior High (both in Piscataway, N.J.).

Subjects. The public schools selected are in adjacent suburban communities with predominantly middle-class population. Children in the Grandview School are assigned to classes (of about 30 children each) so as to include all ability levels in each class. In the case of the third grade classes, the class teacher identified poor-readers and their data were excluded from analysis. The Junior High School students were selected by the school guidance councilor from among children with Otis IQs of 90-110. In the Highland Park school students have a different teacher for each subject-matter class and are assigned to class on the basis of ability level. All classes used were math classes containing students at the middle level of ability. Both public school systems teach "new math": set concepts are introduced in the first grade, intersect in the third, and union at the eighth grade level. The number of boys and girls in each grade are summarized in Table 1. Obviously, it was not possible to treat sex differences as an additional experimental variable.

Insert Table 1 about here.

Procedure. Tests were passed out at the start of the class period and E explained the instructions and defined all words about which S had a question (even ninth graders do not know the meaning of "inanimate"). For the elementary school groups all members of a class received the same test form to facilitate explanation of instructions. With the junior high classes alternate forms were used in each class, partly to control for cheating and partly to control for possible ability differences. All Ss were told there was no time limit: 15-30 minutes seemed to be the usual time range required.

Results

Number of correct responses. A frequency distribution of total correct responses on the 16-item test is given in terms of per cent of each group on each test in Table 2. Group mean and standard deviation appear at the bottom. Interval in which the group median falls is indicated by underlining of the frequency containing the 50% value. There is a good deal of overlap of the distributions for each age group although, in general, the total number of correct answers increases with age. Furthermore, there is a suggestion that the pictorial form of the test is easier than the verbal form, especially for the younger children. Only among the college students are there any substantial numbers of students getting all items correct.

Insert Table 2 about here.

A breakdown of per cent of group correctly responding on each of the test items is summarized in Table 3 for the picture form and in Table 4 for the verbal form. For comparative purposes the data of Nitta and Nagano have been included and are indicated by J following the grade level. Although scores for their second-graders are in some cases lower than for kindergarten children, the discrepancy is undoubtedly attributable to the fact that the test was individually administered to kindergarten children (who don't read) but group administered to second-graders (who presumably do). There is some suggestion, especially at younger ages, that Japanese school children do somewhat better than American school children of comparable grade level. The apparent superiority may be partially attributable to a practice effect among the Japanese children, each of whom did all of the alternate forms. A practice effect was reported despite the fact that no information concerning correctness or incorrectness of the child's answer was ever given. However, the one third-grader of the present study who got 12 of 16 items correct was a girl with a very Japanese name. Conceivably because of language or cultural differences, it may be the case that comprehension of logical connectives develops earlier in Japanese children.

Insert Tables 3 and 4 about here.

Direct examination of the data suggests a number of conclusions: a) Inclusion and exclusion are understood by the majority of Ss at even the youngest ages; there does not seem to be any consistent differential difficulty among items, e.g., A versus $\bar{A}$. b) With the exception of American third-graders, conjunction questions are correctly answered by a majority of Ss at all ages; again, there does not seem to be any consistent differential difficulty among conjunction items. c) The proportion of each age group correctly answering disjunction (union) items increases gradually with age but it is not correctly answered by the majority of the group except at the college-age level. For disjunction, the wording of the item does appear to have an effect. d) Unqualified statements, "A or B", appear to be slightly easier than qualified ones, "A or B or both"; and e) Union of two inclusive, A or B, or two exclusive classes, $\bar{A}$ or $\bar{B}$, is easier than mixed unions, A or $\bar{B}$ and $\bar{A}$ or B. Unfortunately, statistical analyses appropriate for testing these conclusions are not readily attained: age comparisons among all three question types by analysis of variance are inappropriate because of non-homogeneous group variances; X^2 comparisons, which yield more limited information, are limited by extremely low theoretical frequencies of occurrence among the disjunctive items and of non-occurrence among the first eight items. Where legitimate, Chi-square comparisons assuming equal frequency of correct response have been run among the 4 items of a question category for each age group separately as a test of differential item difficulty. The results of these comparisons are indicated in Tables 3 and 4 by an asterisk beside all means whereever the equal frequency assumption may be rejected at or beyond the .05 level. Although there are some significant differences in item difficulty among conjunction questions, especially with the picture version, there does not seem to be any consistent pattern of differential difficulty. For disjunctive questions, on the other hand, A or B seems to be significantly easier than other forms, especially at younger age levels; among older Ss $\bar{A}$ or $\bar{B}$ is about equal to A or B in difficulty, whereas at the college level differential difficulty tends to disappear (at least in the verbal form).

Age groups have been compared by means of analyses of variance for each question category separately (for purposes of these analyses, data of groups 3, 4, 5, 6G, 6H, 7H, 8H and C were employed with random deletion of Ss to achieve equal cell frequencies of N=22). Analysis of total correct responses for third through eighth grade children yield $F(1, 294) = 4.14$ for form, $F(6, 294) = 22.75$ for grade, and $F(6, 294) = 4.18$ for grade x form interaction; all are significant at or beyond the .05 level. In general, the verbal form is more productive of correct response although this difference obtains only for older children; for younger children the pictorial form is somewhat easier. This interaction, along with interactions for analysis of separate question categories, is shown in Figure 1.

Insert Figure 1 about here.

Analysis of number of correct responses on the first four questions yields a significant F for differences between age groups, $F(7, 336) = 10.00$; picture versus words, $F(1, 336) = 19.82$; and form by age interaction, $F(7, 336) = 2.74$; all significant at $p < .01$. These differences remain when college groups are removed from the analysis and Newman-Kuels analysis indicates that they are attributable to third graders performing more poorly than all other groups. The pictorial form is easier for the youngest children but differences between forms disappear in older subjects. For comparison of intersect questions (5-8) only between age group differences are statistically significant: $F(7, 336) = 15.27$ $p < .01$. In this case third graders are significantly below all other groups, college students are significantly better than all other groups, and 4G, 5G, 6G have fewer correct answers than 6H, 7H, 8H. Finally, for the inclusive form of disjunctive questions (questions 9-12) there is a statistically significant effect of age, $F(7, 336) = 36.38$, and age x form, $F(7, 336) = 2.28$. With college groups removed the effect of forms is also significant: $F(1, 294) = 6.25$ (for ages $F(6, 294) = 7.40$; age x form $F(6, 294) = 2.64$). In this instance the verbal form is appreciably easier for intermediate age levels. Newman-Kuels comparisons show that the college students are superior to all other groups. When college student data are removed, eighth graders are superior to all younger groups, 7th graders are superior to 3, 4, and 5 graders and 6th graders are superior to 3 and 4 graders. Thus improvement in performance appears to occur throughout the age range of formal-operation development. For the non-inclusive form of disjunctive items (questions 13-16) the pattern of results is essentially the same with the exception that test form yields a significant main effect whether or not data for college students are included in the analysis ($F(1, 336) = 15.55$ with college data and $F(1, 294) = 23.74$ without). Once again, the pictorial form is somewhat easier for children of intermediate age levels.

Summary. Thus, it would appear that although there are age differences on inclusion and exclusion questions they appear to be directly attributable to poor performance of third graders for whom the test, as a whole, is pretty difficult; it is not surprising that for them the pictorial form is simpler. For conjunction questions there appear to be two age breaks: one between 3rd and 4th graders and a second between the intermediate and junior high grades; form has no effect. For disjunction items there is suggestion of improvement throughout the 6, 7, and 8th grades. Since children within this age range are developing skill at dealing with disjunction, or logical union, it is not surprising that phrasing of question and form of material affect the difficulty of the task. The inclusive form of the question, "A or B or both", is slightly more confusing than the simpler "A or B"; disjunction of homogeneous classes, "A or B" and "$\bar{A}$ or $\bar{B}$", is easier than disjunction of non-homogeneous material and material in verbal form is easier to deal with than material in pictorial form.

Nature of errors. Finally, it is instructive to examine the nature of the errors committed by subjects at each age level; these data are summarized in Table 5 by grade for each form separately with a breakdown fo_ conjunction and the two phrasings of disjunctive items. The first figure in each entry gives the error as a percent of all responses, the second as a proportion of all errors. Errors themselves were classified as probable carelessness (one too few or one too many items circled; all the rest correct); giving conjunction (intersect) for disjunction (union) or vice versa; and giving a single class in place of the required combination (intersect or union). Other, unclassifiable errors are not included.

Insert Table 5 about here.

Data for class inclusion are omitted since there were relatively few errors at all ages and most of these were classifiable as probable carelessness. Although Table 5 looks complicated, it is fairly straight-forward. Errors on conjunctive questions appear to be affected by the form of the material (this was not true for correct responses). Apparently, careless errors are much more likely to arise with verbal material and are much less frequent with pictorial material. With verbal material most errors at all ages seem to arise from creating one or another component class rather than the intersect of two classes. Errors of giving union in place of intersect are relatively infrequent, although they are much more common with pictorial material and appear to increase in frequency with age.

In the case of disjunctive items, careless errors are relatively infrequent, especially on the pictorial form. A major source of error seems to be confusing intersect with union (or logical product with logical sum). Frequency of occurrence of this error varies with phrasing of the question and form of material but, with the possible exception of the youngest children, it seems to be the most common error at all ages. Nitta and Nagano (1963) report similar findings. With the college Ss--and only with them--a new source of error appears. For them there is some tendency to confuse negative and positive, i.e., to give $\overline{A}V\overline{B}$ for AVB and $AV\overline{B}$ for $\overline{A}VB$. On questions 13-16 this error accounts for 20% of all errors on the pictorial form and 30% of all errors on the verbal form.

Discussion

The present experiment repeated part of a much more extensive experiment conducted by Nitta and Nagano (1963) with Tokyo school children. It replicated all of their major findings although, in terms of absolute level of performance, Japanese children appear to be more advanced than American children at the younger age levels. It is difficult to evaluate the cause of the apparent cultural difference.

The form in which material is presented seems to have an effect upon question difficulty. For the test as a whole, verbal instances seem to be more conducive to correct response than pictured ones; however, this order of difficulty is not consistently observed for all types of questions, nor is it independent of age of subject. Nitta and Nagano (1966) also found that verbal form was easier than pictorial which, in turn, was easier than Venn diagrams. They speculated that since this ordering of difficulty seemed to parallel a continuum of specificity (from least to most) perhaps verbal alternatives are more conducive than pictures or diagrams to thinking in terms of a class rather than a specific instance. However, results of a supplementary experiment designed to assess this hypothesis were not clear-cut and in some cases were counter to expectation.

With respect to type of logical operation called for, the present findings again confirm those of Nitta and Nagano. Inclusion and exclusion are correctly performed by a majority of even the youngest children. The conjunction, or logical product, of two classes is also correctly identified by the majority of children in the fourth grade and beyond. There is, however, more evidence of continuing improvement with age than was obtained for inclusion and exclusion. These findings are compatible with data of Inhelder and Piaget (1964) on the development of classification, and support their conclusion that the ability to deal with definition and intersection of classes is attained during the period of concrete operations.

Ability to deal correctly with disjunction (logical summation), on the other hand, seems to be a very late accomplishment. There is little if any evidence of it during the period of concrete operations. The present evidence suggests that this ability develops throughout the period of formal operations and is not fully attained until late adolescence. Additional data on high school students are needed to clarify the course of development throughout this age range. Not surprisingly, in view of the tenuous comprehension of disjunction, specific details of question form and nature of material have a marked effect upon probability of correct response.

Much available evidence shows disjunction to be more difficult than conjunction. Youniss and Furth (1964) report comparable findings for studies of comprehension and of direct tuition (Youniss and Furth, 1967) as does Peel (1967). Studies of concept attainment (e.g., Haygood and Bourne, 1965) consistently report that conjunctive concepts (which are defined by class intersection) are more quickly learned than disjunctive concepts (which are defined by union of classes). Why should this be?

Piaget (Inhelder and Piaget, 1958) asserts that although ability to deal with classes and class intersect is achieved by the end of the period of concrete operations, ability to deal with all 16 binary combinations of classes (of which logical union is an instance) is not attained until late in the period of formal operations. The present data support the contention. Youniss and Furth (1967) attempt to account for the differential difficulty of union and intersect in terms of the number of instances to be included. Although that explanation is partially supported by the present data for intersect (which involves two of eight alternatives) versus union (which involves six alternatives) it would appear to be negated by inclusion and exclusion (which involve four alternatives) which is not more difficult than intersection. This suggests that number and variety of requisite "mental operations" may be the crucial factor. Classification of inclusion and exclusion require only that S focus upon the defining property and scan instances for its presence or absence. Class intersect is slightly more complicated in requiring focus upon two properties and scanning for their conjoint occurrence. No additional operation is required if scanning occurs in parallel; if it is done in series (e.g., scan for A, then rescan all positive instances for B) an additional operation is required and there should be more errors than for simpler classification. The finding of more errors for conjunction than for inclusion-exclusion suggests that scanning is done in series--at least by younger children. The assumption is further supported by the finding that the most common source of error on conjunctive questions is identification of only one class (generally the first-named). For disjunctive questions scanning in series has a high probability of leading to error: selecting alternatives with one property or the other, or with conjoint occurrence of both. This is precisely what most of our school age S do. If S were instructed to cross out inappropriate alternatives (e.g., $\overline{AB}$ for AvB) rather than circle the appropriate ones ($A \cdot B$, $\bar{A} \cdot B$, $A \cdot \bar{B}$) he should do much better. A subsidiary experiment by Nitta and Nagano (1966) shows this to be the case. Despite all this evidence in support of a scanning in series-model, it must be rejected for failure to predict a number of additional findings: the effect of question phrasing and form of material, and differential difficulty among questions (homogeneous unions such as AvB and $\bar{A}$v$\bar{B}$ are easier than heterogeneous ones such as $\bar{A}$vB and Av$\bar{B}$). To encompass these findings one must invoke additional operations for recoding into standard form and storage into short term memory. Such assumptions are intuitively reasonable and have been supported in other experimental contexts (Huttenlocher, 1968; Smedslund, 1968; Trabasso, 1967).

An alternative explanation which merits consideration assumes that children learn the correct meaning of "and" quite early since there is no ambiquity to the word. "Or", on the other hand, is often used in an exclusive sense of "one or the other but not both"; conceivably, S may not know the inclusive interpretation. This explanation is not supported by the error data: it would predict interpretation of product (intersect) as sum (union) to be a very infrequent error whereas, in fact, it is very common. Practically no S at any age gives $A \cdot \bar{B} + \bar{A} \cdot B$ for AvB. Furthermore, although the 8H groups had recently received formal instruction on set union they did not do appreciably better than seventh graders, who had not. Thus analysis of performance in terms of component operations for processing and and storing information seems to provide the most complete explanation of the present data. Elsewhere (Neimark, 1969) I have argued that development of systematic techniques for compressing and organizing information, along with a habitual "set" to perform such processing operations, are the major attainments of formal operations thought.

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2 Since the paper by Nitta and Nagano is not generally available to American readers a brief review of their study is in order. It is an exceptionally extensive and detailed report, even by Japanese standards. Their 10 versions of the test differed in: a) the form in which the set elements are represented (diagrams, pictures, or words); and b) class relations among the A and B sets: disjoint (A and B is a nul set), intersecting (the material used in the present experiment), or inclusive (where B is a subset of A so that AB = B). To give a specific example from the tests with pictorial alternatives: a) for disjoint, A = bird and B = flower and there are 6 elements (birds, fish or flowers, one black and one white); b) intersecting, A = bird and B = black with 8 elements (black and white birds and flowers differing in size); c) inclusive, A = bird and B = black bird with six elements (2 black and 2 white birds differing in size and 2 white flowers). The 10 forms of the test were group administered to

children in the second through eighth grades over 3 or 4 sessions; tests for kindergarten children were individually administered. Form of presentation affected difficulty with Venn diagrams being most difficult and verbal elements easiest. There was no difference in difficulty among the disjoint, intersecting, and inclusive forms. Additional control experiments were run to assess the effect of order of statements on a test and order of administration of tests. Neither had an effect, but performance on later tests--regardless of the form of the test--is better than on earlier tests. Another experiment, using pictures of flags as set elements, required S to circle the applicable elements for one condition, or to X out the inappropriate ones. For logical product (AB) the first condition is easier, while for logical sum (A v B) the second is easier. Since $\overline{AB}$, the negation of a product of two sets, is the union of those sets (i.e., $A \vee B = A\overline{B} + \overline{A}B + AB = \overline{\overline{A} \cap \overline{B}}$; $\overline{A \cap B} = \overline{A} \vee \overline{B}$, etc.), intersect and union can always be interchanged.

Table 1

Composition of Groups

	Pictures			Words		
Grade	N	No. Boys	No. Girls	N	No. Boys	No. Girls
3G	26	15	11	22	9	13
4G	24	12	12	24	12	12
5G	26	12	14	25	12	13
6G	22	12	10	22	12	10
7C	20	15	5	19	0	19
8C	20	7	13	19	14	5
9C	23	7	16	17	13	4
6H	26	11	15	27	9	18
7H	21	11	10	23	12	11
8H	25	13	12	24	13	11
Coll.	29	0	29	29	0	29

Table [illegible]

Frequency distributions of total number correct for each group in % of total N

	Group																					
	Pictures											Words										
No. Cor.	3G	4G	5G	6G	6H	7C	7H	8C	8H	9C	Col.	3G	4G	5G	6G	6H	7C	7H	8C	8H	9C	Col
0												14	4	4								
1												14	4	4								
2	4											0	0	0								
3	15											9	4	4			10					
4	19	25		14	4							9	8	4		4	10				6	
5	27	17	4	9	8					5		18	0	12	9		5				0	
6	12	21	12	32	12	5		9	12	0		4	12	20	18	4	10	4	5.	21.	6	3
7	4	21	31	18	4	5	14	13	12	10		14	21	32	14	22	10	9	32	8	18	
8	12	17	46	9	35	35	57	26	32	25	3	14	42	8	9	15	16	9	21	4	24	
9	4			14	31	10	28	22	28	15	3	4	4	3	9	15	5	13	26	25	18	3
10	0		4	4	4	25	19	13	4	15	7				18	7	5	26	5	17	18	
11	0		4		4	5	5	13	4	20	7			4	9	7	16	4	5	8	6	10
12	4					10	5	4	4	5	21				4	15	5	9	5	4	6	10
13											3				9	4		13		4		17
14							5			5	7			4		7	5	13		4		24
15						5					7									4		24
16									4		41											7
Mean	5.34	6.9	7.5	6.6	7.85	9.35	8.90	9.30	8.60	8.78	13.6	4.27	6.4	6.7	8.5	9.30	7.7	10.39	8.3	9.25	8.5	13.2
S.D.	2.2	1.4	1.2	1.8	1.6	2.0	1.5	1.9	2.1	1.7	2.5	2.8	2.3	2.5	2.4	2.5	3.1	2.3	1.5	2.5	1.8	2.0

Table 3

Percent Correct Responses for each Question for each Group on the Picture Form: J refers to Japanese Data.

Question	1J	2J	3G	4G	4J	5G	6G	6H	6J	7C	7H	8C	8H	8J	9C	Col.	Mean
A	93	66	85	92	88	100	91	100	96	100	95	100	95	95	100	100	94
B	76	45	96	100	63	96	95	96	94	100	90	100	100	81	100	100	90
$\bar{A}$	75	57	58	79	86	96	73	85	94	100	95	96	84	90	85	100	84
$\bar{B}$	86	66	85	100	92	100	100	100	96	95	95	100	100	98	100	100	94
Mean	82	58	81**	93*	82	98-	90-	95-	95	99-	94-	99-	96-	91	96-	100-	90
AB	92	84	35	50	92	50	41	54	98	60	67	61	68	100	65	93	69
$\overline{AB}$	70	43	54	67	84	54	73	85	92	90	100	96	88	83	75	93	78
$\bar{A}B$	81	59	42	92	86	96	68	88	100	95	100	96	96	100	85	97	86
$A\bar{B}$	92	66	38	67	76	54	64	69	98	75	95	83	84	98	80	97	77
Mean	84	63	42	69*	84	64**	62	74*	97	80*	90*	84**	84*	95	76	95-	78
A v B	2	9	15	8	22	19	23	31	59	45	38	39	28	49	55	97	34
$\bar{A}$ v $\bar{B}$	0	0	0	0	0	0	4	0	10	15	0	9	4	17	15	59	8
$\bar{A}$ v B	2	0	0	4	0	0	0	0	12	15	5	4	8	22	20	62	10
A v $\bar{B}$	0	5	4	0	12	4	4	12	41	45	19	30	8	44	35	66	20
Mean	1	4	5-	3-	8	6-	8-	11-	30	30*	16-	20**	12-	33	31*	71**	18
A v B	15	20	12	29	49	31	14	50	82	60	62	48	52	88	60	97	48
$\bar{A}$ v $\bar{B}$	0	0	4	0	0	0	0	0	12	5	5	0	4	15	15	59	7
$\bar{A}$ v B	2	2	0	0	4	4	0	0	14	5	5	0	8	22	5	55	8
A v $\bar{B}$	2	13	4	0	25	12	4	15	65	30	19	39	32	56	50	79	28
Mean	5	9	5-	7-	20	12-	5-	16-	43	25**	22**	22**	24**	45	32**	72*	23

* 2. ** 2.

Table 4

Percent correct response for each question for each group on the Verbal Form: J refers to Japanese Data.

Question	KJ	2J	3G	4G	4J	5G	6G	6H	6J	7C	7H	8C	8H	8J	9C	Col.	Mean
								Group									
A	92	88	64	88	94	84	91	100	98	89	100	95	100	98	88	100	92
B	80	86	36	92	94	88	86	100	100	100	100	100	100	95	94	100	91
$\bar{A}$	80	84	59	79	96	72	86	85	93	68	91	84	67	100	82	83	82
$\bar{B}$	78	75	54	71	98	80	77	89	84	89	83	89	96	90	94	97	84
Mean	82	83	53	82	96	81	85	94^{-}	92	86^{-}	94^{-}	92^{-}	91^{-}	96	90^{-}	95^{-}	87
AB	78	80	36	79	92	72	86	92	98	89	96	95	100	98	88	93	86
$\overline{AB}$	72	50	41	79	73	64	68	89	80	63	83	68	96	81	82	100	74
$\bar{A}B$	78	75	54	58	88	68	68	92	96	74	74	84	83	90	76	97	78
$A\bar{B}$	73	71	64	83	88	64	77	96	96	63	83	100	92	93	94	100	84
Mean	75	69	49	74	85	67	75	92	92	72	84	87^{-}	93	90	85^{-}	98^{-}	80
A v B	17	27	0	4	27	12	27	26	57	21	44	10	17	51	12	28	24
$\bar{A}$ v $\bar{B}$	2	0	0	0	2	4	9	11	25	5	26	5	12	52	6	76	15
$\bar{A}$ v B	2	5	0	0	10	8	4	18	29	10	30	10	17	29	0	86	16
A v $\bar{B}$	2	2	4	0	0	8	23	22	22	10	48	10	17	20	18	76	18
Mean	6	8	1^{-}	1^{-}	10	8^{-}	16^{-}	29	33	12^{-}	37	9^{-}	16	31	9^{-}	66^{**}	18
A v B	17	41	18	12	47	24	64	59	73	37	83	47	63	61	53	69	48
$\overline{A}$ v $\overline{B}$	3	0	0	0	8	8	23	11	22	21	22	15	25	29	18	66	17
$\bar{A}$ v B	7	9	0	0	20	12	32	30	47	21	35	10	17	46	18	79	24
A v $\bar{B}$	3	7	0	0	10	4	36	26	37	16	44	5	46	39	29	72	23
Mean	8	14	4^{-}	3^{-}	21	12^{-}	39^{*}	32^{**}	45	24	46	19	38^{**}	44	30	72	28

Note: * $X^2 \geq 7.8$; ** $X^2 \geq 11.3$, each with 3 d.f.; $^{-}$ X^2 not legitimate

Table 5

Nature oferrors as a percent of all responses and, in parenthesis, as a percent of all errors. See text for classification of errors.

Group	5-8			9-12			13-16		
	I*	II	III	I	II	III	I	II	III
3G	2 (3)	26 (45)	5 (8)	36 (37)	35 (36)	3 (3)	51 (54)	23 (24)	5 (5)
4G	7 (23)	10 (33)	3 (10)	41 (42)	37 (39)	4 (4)	54 (58)	27 (29)	5 (3)
5G	11 (39)	9 (32)	1 (4)	25 (26)	39 (42)	2 (2)	47 (53)	24 (27)	1 (1)
6G	7 (18)	14 (35)	8 (21)	36 (40)	39 (42)	2 (2)	48 (50)	36 (37)	1 (1)
6H	12 (44)	8 (30)	2 (7)	46 (52)	32 (36)	1 (1)	51 (61)	26 (31)	5 (6)
7C	12 (62)	4 (19)	1 (6)	30 (43)	31 (45)	1 (2)	50 (67)	19 (25)	4 (5)
7H	4 (38)	6 (62)	0	42 (49)	34 (41)	0	43 (55)	29 (37)	4 (5)
8C	10 (56)	4 (22)	1 (6)	29 (40)	42 (56)	1 (1)	48 (60	28 (36)	3 (4)
8H	9 (56)	5 (31)	2 (12)	43 (50)	33 (38)	4 (5)	50 (66)	22 (29)	4 (5)
9C	16 (68)	2 (10)	0	28 (40)	34 (49)	2 (4)	40 (59)	24 (35)	2 (4)
Col.	3 (100)	0	0	16 (66)	9 (34)	0	20 (66)	3 (9)	0

Group	5-8			9-12			13-16		
	I	II	III	I	II	III	I	II	III
3G	9 (16)	6 (11)	14 (29)	48 (48)	17 (17)	8 (8)	33 (34)	19 (20)	9 (10)
4G	0	4 (17)	14 (54)	66 (66)	12 (13)	5 (5)	52 (54)	25 (26)	0
5G	7 (21)	8 (24)	9 (27)	52 (56)	12 (13)	7 (8)	28 (32)	38 (43)	8 (9)
6G	3 (14)	9 (30)	6 (23)	36 (43)	24 (28)	15 (18)	20 (33)	11 (18)	18 (30)
6H	0	4 (23)	5 (38)	42 (52)	22 (28)	8 (9)	28 (40)	22 (32)	12 (18)
7C	4 (14)	4 (14)	9 (33)	36 (40)	46 (52)	3 (3)	32 (43)	21 (29)	11 (14)
7H	3 (20)	5 (33)	6 (40)	17 (28)	32 (50)	9 (14)	16 (38)	5 (12)	13 (30)
8C	0	5 (40)	0	63 (70)	16 (17)	4 (4)	37 (46)	20 (25)	7 (8)
8H	0	1 (8)	5 (42)	38 (46)	33 (40)	5 (6)	27 (43)	19 (30)	15 (23)
9C	0	6 (40)	6 (40)	53 (58)	28 (31)	3 (3)	25 (35)	28 (40)	12 (17)
Col.	0	0	2 (67)	22 (64)	6 (18)	3 (10)	6 (18)	4 (15)	7 (9)

*Note: I, intersect as union or vice-versa; II, giving a single class in place of requested combination; III, one too few or one too many.

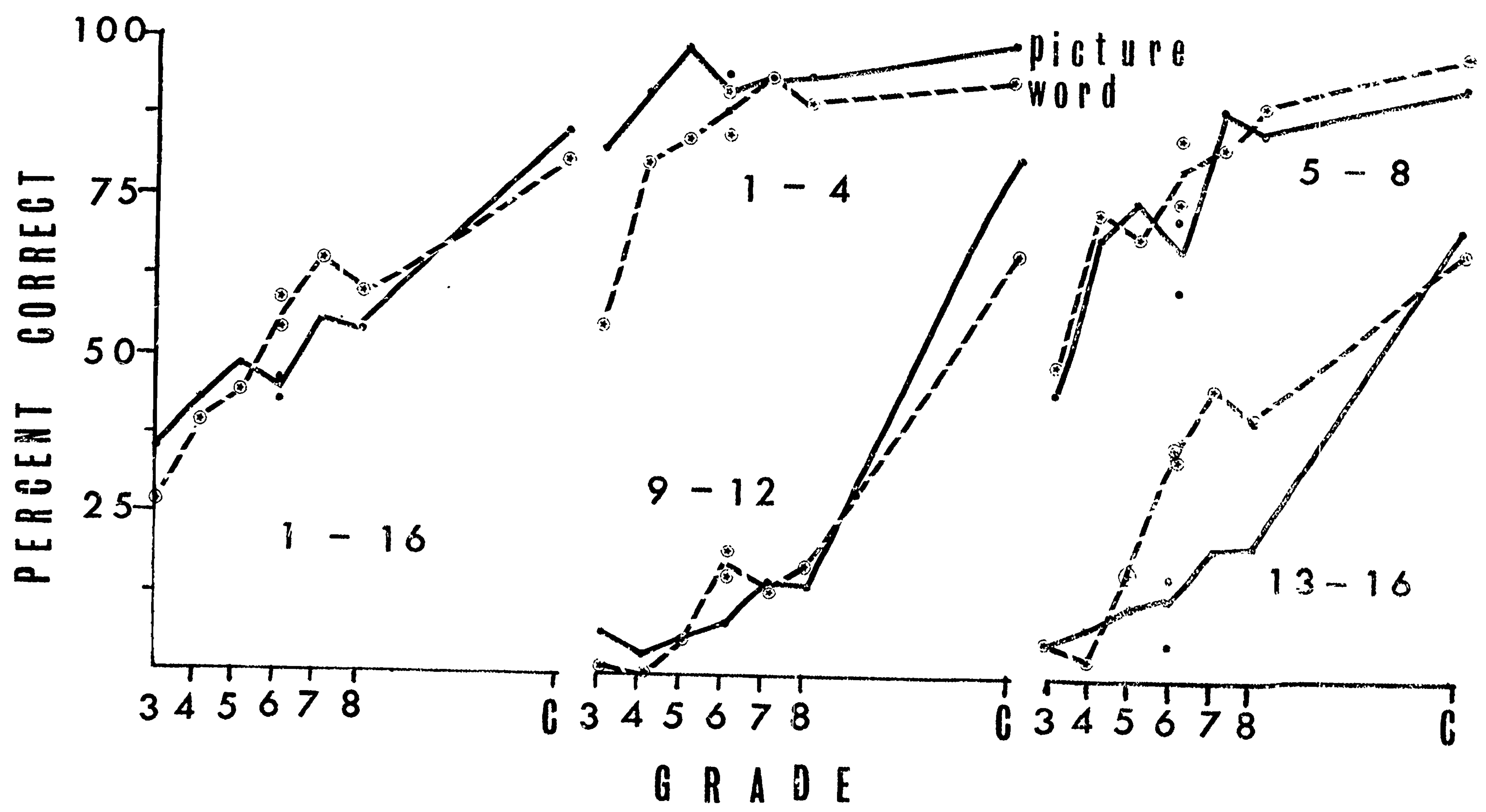

Figure 1 Percent of correct responses as a function of grade level for the test as a whole and for each type of question separately: 1 - 4 are inclusion and exclusion; 5 - 8 class intersect; 9 - 12 class union inclusive form; 13 - 16 class union ambiguous form.